An Open Source model for the simulation of granular flows: First results with GRASS GIS and needs for further investigations

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Abstract

Granular flows like avalanches or debris flows pose a major threat to communities and infrastructures in mountain regions all around the globe. The runout behaviour of granular flows is an important determinant for the degree of hazard connected to such processes. Some physically-based runout models do exist on the market, but they are either expensive or difficult to handle and therefore not widely used. No freely available, user-friendly full capability software for physically-based modelling of granular flows does yet exist.

The major goal of the study presented here is to fill this gap. It was decided to use the Savage-Hutter (SH) model, which is based on a system of differential equations for the conservation of mass and momentum.

A solution developed for simple topographies was implemented into GRASS GIS as a raster module named r.avalanche. The model output was tested for simple artificial topographies as well as for real catchments in the Argentine Andes. The results were promising, but there is need for more research, in particular:

- the present implementation only yields reasonable approximations for simple topographies without pronounced horizontal curvature of the flow channels. An appropriate extension of the Savage-Hutter theory for arbitrary topography has to be worked out and implemented into GRASS;
- the entrainment of particles (snow or soil, respectively) may play a prominent role for the runout length and for the volume deposited and shall therefore be included;
- it shall be taken into account that granular flows are often two-phase flows (solid and liquid fractions).

1 Background

GIS methods play a prominent role in mapping and prediction of landslide, debris flow, and avalanche hazard and risk. They enable an efficient management of spatial data at all scales, usually in raster or vector format. Whilst the GIS market was and still is dominated by proprietary and often expensive software products, the Open Source segment has recently gained a lot of popularity among scientists, national and regional authorities, and companies. A large array of Open Source GIS products, distributed under the GNU license, is available. GRASS – due to its modular design –
allows the straightforward implementation of new modules using the C language and is widely used among natural scientists. Examples of applications of GRASS in the field of natural hazards are the RiskBox including two rock fall simulation algorithms (Cannata 2007) or \textit{r.debrisflow}, a debris flow simulation program (Mergili & Fellin 2007; Mergili et al. in preparation).

GIS methods are widely used with statistical or deterministic models for landslide initiation as well as with empirical-statistical approaches (Vandre, 1985; Rickenmann, 1999) or semi-deterministic models (Gamma, 2000; Wichmann, 2006) for runout of debris flows and snow avalanches. Such methods require a lot of parameter calibration for specific study areas and events, leading to a limited capacity for class A predictions.

Though fully deterministic models for simulating the runout of granular masses date back to the 1950s (Voellmy 1958), GIS implementations of such concepts are still rather scarce. Since the 1980s, various models for the motion of debris flows and flow avalanches have been developed by Savage & Hutter (1989), Takahashi et al (1992), Iverson (1997), or McDougall & Hungr (2004). Particularly the Savage-Hutter (SH) model has gained a lot of attention in the scientific community. The SH model assumes an incompressible fluid moving down an inclined plane, subject to Coulomb bed friction and internal friction. The SH model is based on a system of partial differential equations of mass and momentum balance (compare Eq. [1] to [3] in Chapter 2). The model is scale-invariant, meaning that validation is possible with small-scale experiments as well as with large-scale granular flows in nature.

Solutions for a set of idealized channel topographies have been elaborated (e.g. Pudasaini 2003). Some problems, however, remain unsolved. Among these is the question of entrainment or deposition of solid or fluid, which is a difficult issue due to the complex conditions at the upper and lower boundaries of the flow. Other models (e.g. Iverson 1997; McDougall & Hungr 2004) differ from the SH model in rheological assumptions, geometry, or the details of the numerical implementation (Harbitz 1998). Among all of them, the most detailed comparisons with experiments or field observations have been carried out for the SH model so far.

Chau & Lo (2004) modified the model of Takahashi et al. (1992) to model flow path and deposition of debris flows threatening the Leung King Estate (Hong Kong, China), based on a GIS.

2 Methods

The module \textit{r.avalanche} provides a fully deterministic model for the motion of granular flows like debris flows, snow avalanches, or some types of industrial flows. It was developed as a raster module for GRASS GIS, using the C programming language. A shell script (\textit{r.avalanche.sh}) facilitates data management (input and display). Raster maps and a set of parameters compiled in a text file serve as input. The program code of the present preliminary version as well as a documentation, a detailed manual, and a test dataset can be downloaded from

\url{www.uibk.ac.at/geographie/personal/mergili/scripts.html}.

\textit{r.avalanche} shall become part of the GRASS source tree as soon as the improvements denoted in Chapter 4 have been included.
The model builds on a solution of the SH model for simple concave topographies with an only vertically curved flow path (“talweg”) running out into a horizontal plane, presented by Wang et al (2004) and illustrated in Figure 1. The current release of *r.avalanche* is therefore only suitable for a simple terrain with straight channels. Future development of the model is aimed at overcoming this problem (compare Chapter 4). The solution elaborated by Wang et al (2004) is based on a curvilinear coordinate system aligned with the “talweg”.

![Figure 1: A typical topography suitable for *r.avalanche* (adapted from Wang et al, 2004).](image)

The SH theory is valid for a cohesionless, granular, incompressible material which can be considered as a fluid continuum. It has to be emphasized that all variables are dimensionless, meaning that the model is scale-invariant and can be used for simulating small-scale laboratory tests as well as large-scale flow phenomena in nature. The fundamental equations of the SH model are stated as follows:

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0 \tag{1},
\]

\[
\frac{\partial}{\partial t}(hu) + \frac{\partial}{\partial x}(hu^2) + \frac{\partial}{\partial y}(huv) = hs_x - \frac{\partial}{\partial x}\left(\frac{\beta_x h^2}{2}\right) \tag{2},
\]

\[
\frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}(huv) + \frac{\partial}{\partial y}(hv^2) = hs_y - \frac{\partial}{\partial y}\left(\frac{\beta_y h^2}{2}\right) \tag{3}.\]

where \( h = h(x,y) \) is the avalanche thickness, and \( u \) and \( v \) are the depth-averaged velocities in downslope and cross-slope direction, respectively. \( s_x \) and \( s_y \) are the net driving accelerations, the parameters \( \beta_x \) and \( \beta_y \) are determined by local slope and earth pressure. It is referred to Wang et al (2004) for a description of the geotechnical details of the model and the way how to solve the differential equations for the specific topographic situation illustrated in Figure 1. In particular, the differential equations Eq. [1] to [3] were solved using the NOC (Non-Oscillatory Central Differencing) scheme, which is a numerical scheme that prevents unphysical oscillations.

Furthermore, a slope limiter had to be used in order to restrict the size of the gradients of the flow. In the present study, the so-called minmod limiter, which is known to be the most diffusive one, reducing numerical oscillations, was chosen (compare Wang et al. 2004).
The resulting numerical scheme elaborated by Wang et al (2004), used for the implementation into GRASS, is complex and shall not be presented here. For understanding the GIS-specific aspects of r.avalanche it is sufficient to know that, for each time step, the avalanche thickness \( h \) and the depth-averaged velocities \( u \) and \( v \) are computed for each cell, based on the values of the surrounding 3x3 environment, the previous time step, the boundary conditions (topography, internal friction, bed friction), and additional variables. The simulation is repeated for a user-defined number of time steps. For stability reasons, time step length has to be kept sufficiently small to fulfill the CFL (Courant-Friedrichs-Levy) condition:

\[
\frac{\Delta x}{CFL_x} = \max_{i,j} \left( \frac{|u_{i,j}| + \sqrt{(\beta_x)_{i,j} h_{i,j}}}{\Delta x_{i,j}} \right) \quad \frac{\Delta y}{CFL_y} = \max_{i,j} \left( \frac{|v_{i,j}| + \sqrt{(\beta_y)_{i,j} h_{i,j}}}{\Delta y_{i,j}} \right) [4], [5].
\]

Here, \( \Delta x \) and \( \Delta y \) are the cell sizes in \( x \) and \( y \) direction, and \( i \) and \( j \) are the cell numbers in \( x \) and \( y \) direction. The length of the time step \( \Delta t \) has to be smaller than half the minimum of \( CFL_x \) and \( CFL_y \).

In the model, the length of one time step \( \Delta t \) is determined dynamically, based on the CFL condition from the previous time step.

The SH model and its extension by Wang et al (2004) are not designed for GIS environments and therefore lead to difficulties when being implemented into GRASS. The model uses dimensionless formulations, whereas in GIS it is necessary to use dimensional values. Pudasaini (2003) provided a scaling procedure for solving this problem. The scaling is based on the variables \( L \) (typical avalanche length), and \( H \) (typical avalanche height). A value of 1 m can be assumed for both of them. From now on, the dimensional variables are denoted by a cap.

\[
\hat{x} = L_x \quad \hat{y} = L_y \quad \hat{h} = L_h \quad \hat{u} = u \sqrt{gL} \quad \hat{v} = v \sqrt{gL} \quad \hat{t} = t \sqrt{|L|/g}
\]

where \( g \) is gravity and \( t \) is time.

The major difficulty, however, is that the solution of Wang et al (2004) is based on a curvilinear coordinate system, aligned with a single, horizontally straight “talweg” (the predominant direction of the flow; compare Figure 1). Three steps are required for converting the original rectangular coordinate system in which the input raster maps are provided into the coordinate system to be used for the simulation:

1. the coordinate system is rotated around the \( z \) axis, so that the expected predominant flow path – derived from two user-provided pairs of coordinates in the flow channel – is aligned with the new \( x \) direction;
2. a reference surface is created, defined by the flow path and an inclination of zero in \( y \) (cross-slope) direction;
3. based on this reference surface, the cell size \( \Delta x \) for each \( x \) parallel to the reference surface is computed. The elevation is defined as the height of the terrain perpendicular above the reference surface. This can not be done analytically: an iterative algorithm is required, increasing the shift based on the horizontal surface until tested height and terrain height converge.
3 Results

The module *r.avalanche* was first tested for a simple incline running out in a horizontal plane (Figure 2).

Whilst varying the angle of internal friction $\varphi$ between $32^\circ$ and $37^\circ$ showed only a minor influence on the runout behaviour, a decrease of bed friction angle $\sigma$ from $28^\circ$ to $23^\circ$ led to a significant increase of runout length (Figure 3). These results and the spatial patterns of the motion of the flow are in line with those shown by Wang et al (2004). The unphysical oscillation visible in the Figures 2 and 3 is most probably caused by numerical issues.

The model was then applied to a real study catchment near Mendoza in the Argentine Andes (spatial resolution: 5 m; the DEM was derived by stereo-matching of aerial photos and satellite imagery, the starting areas were mapped from an orthophoto). The runout length was in best accordance to field observations when using a bed friction angle $\sigma = 33^\circ$, which is a rather high value compared to those used for other studies. The simulated patterns of deposition showed a degree of spreading which was not observed in the field (Figure 4), a phenomenon most probably caused by using a too coarse spatial resolution (in order to keep the computation time at an acceptable level).

![Figure 2: Debris flow runout over a simple topography with a bed friction angle $\delta = 28^\circ$.](image)
Figure 3 (top): Longitudinal profiles through a debris flow or avalanche running over a simple topography, comparing the influence of different bed friction angles (23° and 28°).

Figure 4 (bottom): Debris flow runout over a real topography with a bed friction angle $\delta = 33°$. Spatio-temporal distribution of flow height and time series of maximum flow height and velocity. Black lines in the maps indicate the observed delineation of mobilization and entrainment of debris flow material (fine), and of the areas of deposition (bold).
4 Discussion and preview

The first version of \textit{r.avalanche} presented in this paper succeeded in simulating flow phenomena over simple topographies without major changes of flow direction. For more complex topographic conditions, however, it fails. The future steps of program development shall be:

(1) to select and adapt a sound method for modelling rapid granular flows over arbitrary topographies. The method shall be extended by incorporating particle entrainment, which can have important implications for the runout behaviour of granular flows, and the role of pore fluid. The differential formulation of the model shall be derived analytically, using and extending the existing models;

(2) to devise an appropriate numerical scheme (including shock capturing) for solving the equations derived in (1). Numerical solutions of the analytical model for arbitrary topography shall be elaborated and implemented into GRASS GIS, analogous to the present version of \textit{r.avalanche};

(3) to evaluate the quality of the developed approach by comparing it to existing methods and models (DAN, SAMOS) and to validate the results with data from past snow avalanches and debris flows. One particularly interesting case would be the Nomash River Landslide, where McDougall & Hungr (2005) showed the large impact of entrainment on runout. Finding suitable values for the bed friction angle for different types of terrain would be an essential step towards enabling class A predictions with \textit{r.avalanche}.

Furthermore, a more efficient way of data management during the simulation in order to decrease the duration of model execution would facilitate computations at higher spatial resolutions. In the present version, the execution of \textit{r.avalanche} relies on text files and command lines. For the future, a user interface facilitating data management and replacing \textit{r.avalanche.sh} would be desirable.

5 References

Cannata, M 2007, ‘RiskBox: Natural Hazards and Risks Analysis within the GIS GRASS.’ Oral presentation at the FOSS4G conference, Victoria, Canada.


