Landslides starting from unstable slopes threaten people, buildings, and infrastructures all over the world and are therefore intensively studied. On the one hand, engineers use sophisticated models to identify hazardous slopes, mostly based on longitudinal sections. On the other hand, less sophisticated models are used in combination with Geographic Information Systems (GIS) in order to cover larger areas ranging from single slopes to entire countries. The present paper describes an attempt to combine these two philosophies and to come up with a spatially distributed model for slope stability going beyond the widely used infinite slope stability concept.

GIS-supported analyses of slope stability, landslide susceptibility, and landslide hazard have become very common with increased computational power in the last decade [2]. Whilst geostatistical approaches have been applied in some countries (e.g. Italy, Spain, South Korea) in order to get a broad picture of hazardous slopes, deterministic approaches are chosen for more detailed analyses on the small catchment scale. Infinite slope stability models are most commonly employed for determining the factor of safety $FOS$ [1]:

$$FOS = \frac{c + \cos^2 \alpha \gamma_s (d_p - d_w) + (\gamma_s - \gamma_w) d_w}{d_p \gamma_s \sin \alpha \cos \alpha} \tan \phi$$  \hspace{1cm} (1),

where $c$ is the cohesion, $\alpha$ is the slope angle, $\gamma_s$ and $\gamma_w$ are the specific weights of soil and water, $d_p$ is the thickness of soil above the failure plane, $d_w$ is the thickness of saturated soil above the failure plane, and $\phi$ is the angle of internal friction.
Infinite slope stability models work fine for predicting shallow translational slope failures in cohesionless soil on uniform, plane slopes. They are often coupled with models of soil hydraulics.

However, this type of model fails for rotational, deep-seated slope instabilities in cohesive soil or failures of curved or dissected slopes (which can not be seen as „infinite“). Engineers have based their slope stability calculations on circular or elliptical slip surfaces for many years. Traditionally, longitudinal transects (in the direction of the steepest descent) are used. The soil above the slip surface is dissected into a number of columns, and the stabilizing and destabilizing forces are computed for each of them. The summed up values are combined in order to compute the factor of safety. For circular slip surfaces, the forces between the columns can be neglected. For elliptical surfaces, they are often neglected, too, since this simplifies the computation considerably and, even so, leads to a reasonable approximation. Monte-Carlo approaches are frequently used for identifying the most critical slip surface for the area under investigation [3, 4].

Using longitudinal sections means that the real topography of a slope is not accounted for. In some cases, this may lead to severe misinterpretations of the slope stability status. Some few attempts are documented to overcome this problem by combining the approach described above with GIS, for example the work of Xie et al. [3, 4].

The work presented here shows an attempt to integrate an advanced slope stability model based on an ellipsoidal slip surface with a raster-based Open Source Geographic Information System (GRASS GIS), in order to allow for spatially distributed analysis of slope stability going beyond the widely used infinite slope stability concept.

The program requires a digital elevation model, the slope parameters (cohesion, angle of internal friction, specific weight), for layered slopes the depth of each layer, and the ground water level as input.

With these parameters, the slope stability computation is run for a user-defined number of times, each time using an ellipsoid with randomly determined geometrical parameters. The longest axis of the ellipsoid (a) follows the steepest descent of the slope, the shortest axis (c) is aligned normal to the terrain surface (Fig. 1). Minimum and maximum lengths of the axes as well as the offset of the centre over the terrain are defined by the user in order to constrain the randomization and to avoid unrealistic values. After transforming the ellipsoid into the GIS coordinate system (rotation and tilting, compare [3]), the depth of
the slip surface is determined for each raster cell. In contrast to many other applications of this type of model, the slip surface is not necessarily the surface of the ellipsoid, but also weak layers within the ellipsoid are considered (compare Fig. 1 and 2).

![Fig. 1. Ground plot and longitudinal section of the ellipsoidal slip surface. Note that slips are also possible within the ellipsoid along weak layers.](image)

The stabilizing and destabilizing forces are then computed for each raster cell, and the factor of safety is derived according to the revised Hovland’s model [3, 4]:

\[
FOS = \frac{\sum [cA + (W \cos \theta - U)\tan \phi] \cos \theta_{av}}{\sum W \sin \theta_{av} \cos \theta_{av}}
\] (2),

where \( A \) is the area of the slip surface of the cell, \( W \) is the weight of the overlying soil, \( U \) is the pore water force, \( \theta \) is the angle of the slip surface, and \( \theta_{av} \) is the average inclination of the slip surface along the steepest descent. For detailed information how to obtain these parameters please consult [3] and [4]. Additional forces or seismic loads are not considered.

After repeating the slope stability computation for each randomly determined ellipsoid, the minimum factor of safety from the overlay of all ellipsoids is determined for each raster cell as well as the lowermost slip surface with \( FOS < 1 \).

Fig. 2 shows the tremendous influence of the choice of the slip surface, using a simple imaginary topography with frictionless soil and a weak layer at 2.5 m depth, bedded on stable rock. The ellipsoid (which is a circle in this example) as slip surface reaches far into the bedrock, resulting in stable conditions \( (FOS = 7.87) \). Allowing the soil to fail at the bottom of the weak layer leads to unstable conditions \( (FOS = 0.81) \).
The model was then run for a terrain from the real world. Information on soil parameters \((c, \varphi, \text{layering})\) was limited. The parameters were therefore calibrated in a way that the factor of safety computed for the very steepest slopes was around 1. The model was executed for 100,000 ellipsoids, assuming dry soil. The results were compared to those from an infinite slope stability model. The same procedure was repeated with fully saturated soil (Fig. 3). For the infinite slope stability model, the potential failure plane was set to the maximum of the \(c\) half axis of the ellipsoids (25 m).

For dry conditions, the general patterns of both maps are the same. However, the factor of safety derived from the infinite slope stability model \((\text{FOS}_{\text{infin}})\) varies much more over small areas than that based on the advanced model \((\text{FOS}_{\text{adv}})\). This is not surprising since in the infinite slope stability model, each raster cell is considered individually, whilst the values of \(\text{FOS}_{\text{adv}}\) are the minima from calculations over larger areas. This implies also that the minima of \(\text{FOS}_{\text{infin}}\) were smaller than those of \(\text{FOS}_{\text{adv}}\). However, averaging out of the values led to larger areas identified as unstable based on the ellipsoidal slip surfaces.

Slope stability decreased considerably when assuming fully saturated soil. \(\text{FOS}_{\text{adv}}\) was far below 1 over much of the steeper portions of the valley slopes. In general, the same was true for \(\text{FOS}_{\text{infin}}\), though the values were usually higher than those of \(\text{FOS}_{\text{adv}}\).

It can be concluded that the presented integration of an advanced slope stability model with GIS yields plausible results for real topographies and may become a valuable tool regarding hazard analysis. However, further evaluation is required, particularly tests with geotechnical parameters from the real world, the optimization of the parameters for the ellipsoid, and an assessment under which conditions such an advanced model is required and where the application of an infinite slope stability model is sufficient.
Fig. 3. Factor of safety for real topography under dry and saturated conditions, computed with ellipsoidal slip surface and with an infinite slope stability model

REFERENCES


